Ralf Schützhold

Department of Physics and Astronomy, University of British Columbia, Vancouver B.C., V6T 1Z1 Canada

Electronic address: `schuetz@physics.ubc.ca`

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According to recent astrophysical observations the large scale mean pressure of our present universe is negative suggesting a positive cosmological constant like term. This article addresses the question of whether non-perturbative effects of self-interacting quantum fields in curved space-times may yield a significant contribution. Focusing on the trace anomaly of quantum chromo-dynamics (QCD), a preliminary estimate of the expected order of magnitude yields a remarkable coincidence with the empirical data, indicating the potential relevance of this effect.

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Recent measurements of the cosmic microwave background [1] suggest that the large scale structure of our universe is quite accurately described by the conformally flat Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = d\tau^2 - \Omega^2(\tau)\, d\boldsymbol{r}^2 = \Omega^2(t)\left(dt^2 - d\boldsymbol{r}^2\right)\,, \tag{1}$$

with τ denoting the proper and t the conformal time, respectively. The temporal variation of the scale factor Ω^2 inducing the cosmological red-shift is represented by the Hubble parameter

$$\mathfrak{H} = \frac{1}{\Omega}\frac{d\Omega}{d\tau} \approx 10^{-10}\text{year}^{-1}\,. \tag{2}$$

Inserting the FRW metric in Eq. (1) into the Einstein equations (with the cosmological constant λ)

$$\mathfrak{R}_{\mu\nu} - \frac{1}{2}\mathfrak{g}_{\mu\nu}\mathfrak{R} = \left(8\pi\, G_{\rm N}\langle\hat{T}_{\mu\nu}\rangle - \mathfrak{g}_{\mu\nu}\lambda\right)_{\rm ren}\,, \tag{3}$$

the aforementioned observations and supernova [2] data consistently yield the following conclusions: the 00-component of the r.h.s. of Eq. (3) equals (at least approximately) the critical density $\varrho \approx \varrho_{\rm crit}$ and the spatial *ii*-components – associated with the pressure p – are negative: $p/\varrho_{\rm crit} \approx -2/3$. As a result the universe is presently undergoing an accelerated expansion as approximately described by the de Sitter metric $\Omega(\tau) = \exp\{\mathfrak{H}\tau\}$.

A negative pressure together with a positive energy density necessarily implies a non-vanishing trace of the energy-momentum tensor $T_{\mu\nu}$. Since $T_{\mu\nu}$ can be derived via the variation of the action $\mathcal{A}$ with respect to the metric $\mathfrak{g}^{\mu\nu}$, i.e. $T_{\mu\nu} = 2(-\mathfrak{g})^{-1/2}\delta\mathcal{A}/\delta\mathfrak{g}^{\mu\nu}$, its trace corresponds to the change of $\mathcal{A}$ under the conformal transformations $\mathfrak{g}_{\mu\nu}(\underline{x}) \to \Omega^2(\underline{x})\mathfrak{g}_{\mu\nu}(\underline{x})$ by virtue of Euler's law: $T^\rho_\rho = -\Omega(-\mathfrak{g})^{-1/2}\delta\mathcal{A}/\delta\Omega$, see e.g. [3].

Let us focus on the contribution of the $SU(3)$-color gauge field theory of QCD to the r.h.s. of the Einstein equations (3) in the following. Its dynamics are governed by the well-known Lagrangian density

$$\mathcal{L} = -\frac{1}{4}\,G^a_{\mu\nu}G_a^{\mu\nu} + \bar\psi\left(i\gamma^\mu\partial_\mu + g\,T_a\gamma^\mu A^a_\mu - m\right)\psi\,, \tag{4}$$

with $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a{}_{bc}A^b_\mu A^c_\nu$ being the gluonic field strength tensor. Here $f^a{}_{bc}$ denote the $SU(3)$ structure constants, T_a its fundamental generators, and g is the strong coupling. For simplicity we drop the ghost fields as well as the gauge fixing terms and consider only one single flavor, i.e. quark-species ψ. The remaining electro-weak sector of the standard model will be discussed at the end of this article.

On the classical level all gauge field theories as described by Eq. (4) are conformally invariant (for $m = 0$). According to the above arguments this feature implies a vanishing trace of the classical energy-momentum tensor (as one can easily check by an explicit calculation.) Turning to the quantum field theoretical description the situation becomes more complicated. In the first place, the naïve expectation value of the operator-valued energy-momentum tensor diverges due to the infinite zero-point energy. In order to renormalize this singularity by an appropriate counter-term one has to interpret the cosmological constant λ in Eq. (3) as a bare quantity [3].

After such a minimal subtraction procedure the trace of the renormalized expectation value of the energy-momentum tensor $\langle\hat{T}_{\mu\nu}\rangle_{\rm ren}$ vanishes (again assuming $m = 0$) for free ($g = 0$) fields in flat ($\Omega = 1$) space-times – but *not* in the general case. This phenomenon is called the trace anomaly and goes along with the dynamical breaking of the conformal invariance of the classical theory in Eq. (4). It has been calculated for two limiting cases: firstly for self-interacting quantum fields in flat ($\Omega = 1$) space-times [4] and secondly for free ($g = 0$) fields in curved space-times [3]. In the first case one obtains [4]

$$\langle\hat{T}^\rho_\rho\rangle_{\rm ren} = \frac{\beta(g)}{2g}\langle\hat{G}^a_{\mu\nu}\hat{G}_a^{\mu\nu}\rangle_{\rm ren} + (1+\gamma^m)\,m\,\langle\bar\psi\hat\psi\rangle_{\rm ren}\,. \tag{5}$$

The Callan-Symanzik or Gell-Mann–Low β-function describes the scale dependence $\beta = \mu\,\partial g/\partial\mu$ of the renormalized coupling $g(\mu)$ and reflects the dynamical breaking of the conformal invariance of the classical theory (dimensional transmutation). Similarly, the γ^m-function corresponds to the running of the renormalized mass. The expectation values $\langle\hat{G}^a_{\mu\nu}\hat{G}_a^{\mu\nu}\rangle_{\rm ren}$ and $\langle\bar\psi\hat\psi\rangle_{\rm ren}$ occurring in Eq. (5) represent the so-called gluonic and quark condensates, respectively, see e.g. [5]. These inherently non-perturbative quantities again reflect the dynamical breaking of the classical scale invariance. Both are of great experimental relevance and their values have been confirmed within several contexts, see e.g. [5]. Since the symmetry breaking scale $\Lambda_{\rm QCD}$ is (for $m=0$) the only scale in the theory (4), it yields $\langle\hat{G}^a_{\mu\nu}\hat{G}_a^{\mu\nu}\rangle_{\rm ren}=\mathcal{O}(\Lambda^4_{\rm QCD})$ and $\langle\bar{\hat\psi}\hat\psi\rangle_{\rm ren}=\mathcal{O}(\Lambda^3_{\rm QCD})$ – at least for $m\ll\Lambda_{\rm QCD}$. As it is well-known, the β-function occurring in Eq. (5) can be calculated within the framework of perturbation theory and it turns out to be negative. Consequently it is now commonly accepted (cf. [5]) that the QCD trace anomaly gives rise to a negative energy density (since $\langle\hat{G}^a_{\mu\nu}\hat{G}_a^{\mu\nu}\rangle_{\rm ren}>0$ and $\langle\bar{\hat\psi}\hat\psi\rangle_{\rm ren}<0$) of the QCD vacuum in the Minkowski space-time.

However, such a huge amount of negative energy density of order $\mathcal{O}(\Lambda^4_{\rm QCD})$ blatantly contravenes our observations. This drastic and global violation of the (weak and dominant) energy conditions (see e.g. [3]) in the Minkowski space-time goes along with a fundamental contradiction if one includes gravity since the r.h.s. of the Einstein equations (3) associated with a flat space-time vanishes.

Consequently, regarding the Einstein equations (3), one is led to absorb the aforementioned energy density by renormalizing the cosmological constant λ, in complete analogy with the case of the zero-point energy (which determines the divergent part of λ only). In the same manner as one adjusts the mass-counter term in the self-energy renormalization of the electron, for example, one has to fix the bare cosmological constant by demanding that the r.h.s. of the Einstein equations (3) vanishes for the Minkowski vacuum

$$\left(8\pi\, G_{\rm N}\langle\hat{T}_{\mu\nu}\rangle-\mathfrak{g}_{\mu\nu}\lambda\right)^{\text{Minkowski vacuum}}_{\rm ren}=0\,. \tag{6}$$

On the other hand, a non-trivial geometry of the space-time may also induce a non-vanishing trace – even for free fields (second limiting case $g=0$). In this case $\langle\hat{T}^\rho_\rho\rangle_{\rm ren}$ is given by the sum of a bilinear form of the curvature tensor (such as $\mathfrak{R}_{\mu\nu}\mathfrak{R}^{\mu\nu}$ or $\mathfrak{R}^2$) and second derivatives of it ($\Box\mathfrak{R}$), cf. [3]. For the free QCD field (with $g=m=0$) within the de Sitter space-time $\Omega(\tau)=\exp\{\mathfrak{H}\tau\}$, for example, one finds ($\hbar=c=1$)

$$\langle\hat{T}^\rho_\rho\rangle_{\rm ren}=\frac{281}{120\pi^2}\mathfrak{H}^4\,. \tag{7}$$

In contrast to the contribution in Eq. (5), there is no reason to absorb this term by renormalization of λ. In view of its potentially space-time dependent character such a procedure would be rather strange. However, here the associated energy density is far too small to explain the observations [1,2].

In summary, the (renormalized) expectation value of the energy-momentum tensor acquires an anomalous trace for self-interacting quantum fields in flat space-times (5) on the one hand as well as for free fields in curved space-times (7) on the other hand. However, both effects taken alone are not capable of explaining the negative pressure as suggested by the observations [1,2]. But this is just what one might expect, since realistic investigations have to involve both contributions simultaneously, i.e. the (non-perturbative) effects of self-interacting fields in curved space-times. A rigorous derivation of the renormalized expectation value of the energy-momentum tensor for this scenario appears to be rather involved and is not the aim of the present article. Here we just give a preliminary estimate of the expected order of magnitude of the effect. To this end we employ an adiabatic approximation (cf. [3]) by exploiting the huge difference of the involved time scales. The cosmic evolution – governed by $\mathfrak{H}$ – is extremely slow compared to the typical fluctuations of the quantum field as determined by $\Lambda_{\rm QCD}$. Consequently the adiabatic approximation is an expansion in the small parameter $\mathfrak{H}/\Lambda_{\rm QCD}=\mathcal{O}(10^{-40})$. So the zeroth-order term is the pure Minkowski (flat space-time) contribution whereas the first-order term represents the lowest correction induced by the cosmic expansion.

In order to calculate the renormalized expectation value of the energy-momentum tensor it is essential to specify the correct vacuum state associated with our expanding universe, cf. [3]. To this end we adopt the Schrödinger picture

$$\frac{d}{dt}|\Psi\rangle=-i\hat{H}_{\rm FRW}(t)|\Psi\rangle\,, \tag{8}$$

where $\hat{H}_{\rm FRW}(t)$ denotes the Hamilton operator, i.e. the generator of the time evolution, with respect to the conformal coordinates $(t,\boldsymbol{r})$ in Eq. (1). Within the adiabatic approximation, the explicitly time-dependent Hamiltonian $\hat{H}_{\rm FRW}(t)$ of an expanding universe can be related to the (time-independent) Minkowski Hamiltonian $\hat{H}_{\rm Min}$ via

$$\hat{H}_{\rm FRW}(t)=\exp\left\{-i\Omega(t)\hat{S}\right\}\hat{H}_{\rm Min}\exp\left\{+i\Omega(t)\hat{S}\right\}\,, \tag{9}$$

with $\hat{S}$ being the generator for the conformal transformations $\mathfrak{g}_{\mu\nu}\to\Omega^2\mathfrak{g}_{\mu\nu}$ in the Schrödinger picture. In terms of a dynamically scaled state defined via $|\widetilde\Psi\rangle=\exp\{+i\Omega(t)\hat{S}\}|\Psi\rangle$, the Schrödinger equation assumes the form

$$\frac{d}{dt}|\widetilde\Psi\rangle=-i\left(\hat{H}_{\rm Min}-\dot\Omega\hat{S}\right)|\widetilde\Psi\rangle\,. \tag{10}$$

Treating $\hat{H}_1=-\dot\Omega\hat{S}=\mathcal{O}(\mathfrak{H})$ as a perturbation and switching to the interaction representation $\hat{S}(t)=\exp\{+i\hat{H}_{\rm Min}t\}\,\hat{S}\,\exp\{-i\hat{H}_{\rm Min}t\}$ we may solve the above equation in linear response, i.e. first order adiabatic expansion

$$|\widetilde\Psi\rangle=|\widetilde\Psi_{\rm in}\rangle+i\int\limits_{-\infty}^{0}dt\,\dot\Omega(t)\,\hat{S}(t)\,|\widetilde\Psi_{\rm in}\rangle+\mathcal{O}(\mathfrak{H}^2)\,. \tag{11}$$

If we assume $\Omega(t)=\Omega_0+\exp\{\mathfrak{H}t\}$, it is reasonable to take the Minkowski vacuum $|0_{\rm Min}\rangle$ with $\hat{H}_{\rm Min}|0_{\rm Min}\rangle=0$

as the initial condition $|\widetilde{\Psi}_{\rm in}\rangle = |0_{\rm Min}\rangle$. The remaining time integration yields $\hat{H}_{\rm Min}^{-1}$ and hence we arrive at

$$|0_{\rm FRW}\rangle = |0_{\rm Min}\rangle + \mathfrak{H}\,\hat{H}_{\rm Min}^{-1}\,\hat{S}\,|0_{\rm Min}\rangle + \mathcal{O}(\mathfrak{H}^2)\,. \tag{12}$$

Therefore the adiabatic QCD vacuum $|0_{\rm FRW}\rangle$ of an expanding universe is *not* the instantaneous ground-state $|0_{\rm Min}\rangle$ of $\hat{H}_{\rm FRW}(t)$ or $\hat{H}_{\rm Min}$ – it acquires corrections already in the first order of the adiabatic expansion. Instead it is the ground-state of the corrected Hamiltonian $\hat{H}_{\rm Min} - \dot{\Omega}\hat{S}$, which can easily be verified using stationary perturbation theory (see e.g. [3] for free fields).

The remaining question is, of course, whether the first-order correction $\mathfrak{H}\,\hat{H}_{\rm Min}^{-1}\,\hat{S}\,|0_{\rm Min}\rangle$ to the vacuum state entails a first-order correction to the expectation value of $\hat{T}_{\mu\nu}$. In order to illustrate this point let us consider the simple example of a time-dependent harmonic oscillator

$$\hat{H}(t) = \frac{\omega}{2\Omega(t)}\left(\hat{P}^2 + \Omega^2(t)\hat{Q}^2\right)\,, \tag{13}$$

where ω corresponds to $\Lambda_{\rm QCD}$. In examining the question of whether the dynamical scale symmetry breakdown in QCD can be modeled by such a simple quadratic potential one might consider the $CP(N-1)$ or the $O(N)$ σ-models [6]. These strongly interacting theories reproduce several features of QCD, such as dynamical scale symmetry breakdown. They can be solved in the large N-limit and in the leading order they effectively behave like massive free fields. After a normal mode decomposition one therefore indeed obtains terms like the one above. For the Hamiltonian in Eq. (13) a change of the scale factor $\Omega(t)$ as in Eq. (9) can simply be generated by the squeezing operator

$$\hat{S} = \frac{1}{4}\left\{\hat{P},\hat{Q}\right\} = \frac{i}{4}\left[\left(\hat{a}^\dagger\right)^2 - \left(\hat{a}\right)^2\right]\,. \tag{14}$$

Consequently the expectation values of operators such as $\hat{Q}^2$ or $\hat{P}^2$ do not acquire a first-order correction. This result can be transfered directly to free quantum fields: pitching on a particular normal mode with the wavelength k the conformal charge $\hat{S}$ again acts like a squeezing operator $\hat{S} \to i(\hat{a}_k^\dagger\hat{a}_{-k}^\dagger - \hat{a}_k\hat{a}_{-k})$. As a result there is no first-order correction to the expectation value of $\hat{T}_{\mu\nu}$ for free (linear) fields.

However, if we leave the free-field sector and take interactions into account the situation may change: let us consider the interaction Hamiltonian

$$\hat{H}_{\rm int}(t) = g\int\limits_{-\infty}^{t} dt'\,G_{\rm ret}(t-t')\left\{\hat{Q}^2(t),\hat{Q}^2(t')\right\}\,, \tag{15}$$

where $\hat{Q}(t) = \hat{Q}\cos(\omega t) + \hat{P}\sin(\omega t)/\omega$ denotes the unperturbed time-dependent operator in the interaction picture. The retarded propagator $G_{\rm ret}(t-t')$ encodes the dynamics of an intermediate (interaction) degree of freedom which has been integrated out. Again such a term can be motivated by the $CP(N-1)$-models: in the large N-limit these strongly interacting massless theories effectively transform into massive fields obeying weak (i.e. next-to-leading order in $1/N$) long-range four-point interactions, cf. [6].

For general Green functions $G_{\rm ret}(t-t')$, the operator in Eq. (15) does entail a first-order correction. This can be most easily verified by assuming $g \ll 1$ which allows for a perturbative treatment.

In view of these considerations one might expect a first-order contribution to $\langle\hat{T}_{\mu\nu}\rangle_{\rm ren}$ to be possible in the case of QCD: since the classical as well as the free quantum field in Eq. (4) are (for $m=0$) conformally invariant, their solutions would simply be scaled during the expansion of the universe – like the red-shift of the photon field. (This would actually happen if the universe were to expand very rapidly $\mathfrak{H} \gg \Lambda_{\rm QCD}$.) However, the strong self-interaction on the quantum level breaks the conformal invariance and introduces a fixed scale $\Lambda_{\rm QCD}$ leading to a positive pressure given by Eq. (5). Within an expanding universe the balance of these two tendencies, i.e. following the expansion on the one hand and retaining the scale on the other hand, leads to a displaced vacuum state (12).

Let us assume that a part of the positive vacuum pressure in Eq. (5) can be explained by relatively localized (non-perturbative) vacuum fluctuations (e.g. instantons [7] or oscillons [8]) which repel each other (at least in average, cf. [7]). Let us further assume that the dynamical breakdown of the scale symmetry is basically encoded by these (non-perturbative) vacuum-fluctuations whereas their (repulsive) interactions are adequately described by the free (perturbative) and thus conformally invariant field equations. In this case their solutions would simply be scaled during the cosmic expansion in contrast to the non-perturbative fluctuations which retain their scale and hence are not affected. Within an expanding universe, then, every vacuum fluctuation "sees" all other vacuum-fluctuations "red-shifted", i.e. their repulsion acquires a correction proportional to $\mathfrak{H}R$, where R denotes their (mean) distance (cf. [7]). Accordingly, the positive vacuum pressure in Eq. (5) gets diminished by an amount of first order in $\mathfrak{H}$.

Based on this intuitive picture it appears plausible to admit a correction to the expectation value of the energy-momentum tensor within the FRW vacuum in Eq. (12) which is linear in $\mathfrak{H}$. After the renormalization described in Eq. (6), i.e. the subtraction of the Minkowski contribution, we therefore obtain

$$\begin{aligned}\langle\hat{T}_{\mu\nu}\rangle_{\rm ren}^{\rm FRW} &= \mathfrak{H}\langle\,\hat{T}_{\mu\nu}\,\hat{H}_{\rm Min}^{-1}\,\hat{S}\,\rangle_{\rm ren}^{\rm Min} + \text{H.c.} + \mathcal{O}(\mathfrak{H}^2)\\ &= \mathcal{O}(\mathfrak{H}\Lambda_{\rm QCD}^3)\,.\end{aligned} \tag{16}$$

Let us estimate the associated order of magnitude: although $\Lambda_{\rm QCD}$ depends on the renormalization scheme we may fix it approximately via $\Lambda_{\rm QCD} = \mathcal{O}(10^8\,{\rm eV}) = \mathcal{O}(10^{14}\,{\rm m}^{-1})$. The masses of the light quarks which dominantly couple to the gluonic field are roughly of a similar order of magnitude. The Hubble expansion parameter $\mathfrak{H}$ is about $10^{-26}\,\mathrm{m}^{-1}$. Inserting the above values we finally arrive at $\langle\hat{T}_{\mu\nu}\rangle^{\mathrm{FRW}}_{\mathrm{ren}} = \mathcal{O}(10^{16}\,\mathrm{m}^{-4})$ or $\langle\hat{T}_{\mu\nu}\rangle^{\mathrm{FRW}}_{\mathrm{ren}} = \mathcal{O}(10^{-29}\,\mathrm{g\,cm}^{-3})$. By inspection one finds that the deduced order of magnitude nicely fits the empirical data $\varrho_{\mathrm{crit}} \approx 10^{-29}\,\mathrm{gcm}^{-3}$. In view of the huge difference of the involved scales $[\mathfrak{H}/\Lambda_{\mathrm{QCD}} = \mathcal{O}(10^{-40})]$ this remarkable coincidence seems to be almost too good to be just an accident. At least it indicates the potential relevance of the effect described in the present article with regard to the interpretation of the astrophysical data [1,2].

It should be mentioned here that a pressure induced by the expansion of our universe with $p \propto \mathfrak{H}$ generates a cosmic evolution which differs from that with a true cosmological constant $p = \mathrm{const}$: by inserting the FRW metric (1) into the Einstein equations (3) one obtains the Friedmann equation $3\mathfrak{H}^2 = 8\pi\, G_{\mathrm{N}}\varrho$. Furthermore the Einstein equations imply $\nabla_\mu\langle\hat{T}^{\mu\nu}\rangle_{\mathrm{ren}} = 0$, i.e. $d\varrho/dt = -3(\varrho+p)\mathfrak{H}$. Combining these two equalities and specifying the pressure p one may determine the time-evolution of our universe. Unfortunately the presently available data (such as the Hubble parameter or the age of the universe) are not precise enough to distinguish the two cases ($p \propto \mathfrak{H}$ and $p = \mathrm{const}$).

Of course one may ask whether the remaining electroweak sector of the standard model generates similar contributions: typically (see e.g. [5]) non-perturbative effects (such as $\langle\hat{G}^a_{\mu\nu}\hat{G}^{\mu\nu}_a\rangle_{\mathrm{ren}}$ and $\langle\bar{\hat{\psi}}\hat{\psi}\rangle_{\mathrm{ren}}$) display a dependence on the coupling of $\exp\{-8\pi^2/g^2\} = \exp\{-2\pi/\alpha\}$. The scale of the dynamical symmetry breaking Λ_{QCD} obeys a similar non-analytical dependence on the coupling g. Inserting $\alpha_{\mathrm{QED}} \approx 1/137$ into the above expression one obtains a suppression by an order of magnitude of 10^{-370}. Hence the contributions arising from the dynamical breaking of scale invariance can safely be neglected in this case.

The remaining explicit breaking of the scale symmetry induced by the Higgs field of course also generates contributions to $\hat{T}_{\mu\nu}$. However, the general structure of all these terms is given by $m^2\,\langle\hat{\Phi}^\dagger\hat{\Phi}\rangle_{\mathrm{ren}}$ and according to the arguments after Eq. (14) they do not contribute to the first order in $\mathfrak{H}$. Although the mixture of these terms caused by interactions remains subject to further considerations, a contribution of the electro-weak sector in analogy to QCD is not obvious.

In summary, the present article motivates a deeper examination of the vacuum of strongly interacting fields in the gravitational background of our expanding universe – for the present epoch as well as for earlier stages, cf. [9]. These investigations might perhaps lead to a better understanding of some of the problems in cosmology without necessarily invoking yet unknown low-energy fields, for example quintessence (see e.g. [10]).

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